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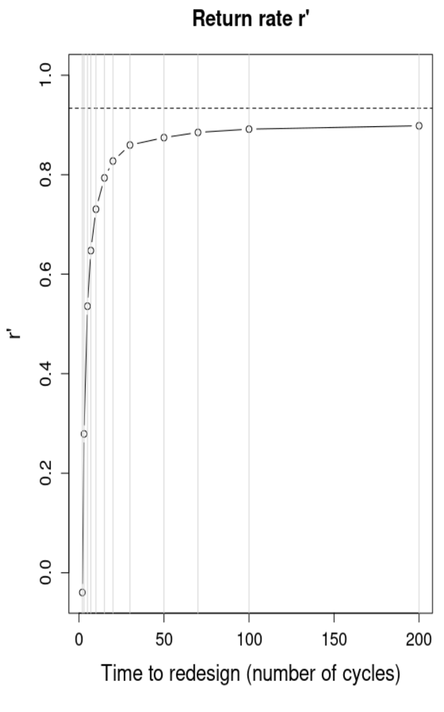
Note

A reuse system for bottles – trip rate calculations under model replacement



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Date 23 October 2023

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Abstract

The back to market trip rate is a key performance indicator of a bottle reuse system. In this study we investigate the impact of model replacement on the trip rate, incorporating also other loss mechanisms: incomplete deposit, damage or pollution, and scuffing.

We find that for representative market parameter values, the overall trip rate is significantly reduced from introducing the scuffing and model replacement components into a system originally comprising the deposit and damage loss processes only, with figures in the range 36% - 57% for PET bottles, and 32% - 64% for glass bottles. The effect increases for shorter model design lifetimes.

Keywords	Reuse system, back to market, trip rate, return rate, model replacement, simulation
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1 Introduction

A key performance indicator of a reuse system for bottles is the system's trip rate. This attribute quantifies the average number of servings delivered for each unit produced in the system. The rate will depend on the return and waste mechanisms incorporated into the system. In general, every return process that is included and modelled by the system, at the same time leaves a discharge point for bottles, thus reducing the overall trip rate of the system. In this study, we pay particular interest to the effect of model replacement on the trip rate.

2 Problem

We consider a reuse system that initially holds a full stock of n_{total} new bottles. The system comprises four separate processes where bottles are possibly leaked. Leaked bottles are replaced by freshly produced bottles from outside the system so that the total number of bottles in stock is maintained.

The first leakage process represents the deposit loss component, ie empty bottles not being returned by the consumer. This is considered to act randomly among the bottles circulating the reuse system. Next, a certain share of the bottles deposited are damaged or polluted, and thus taken out from further circulation. This mechanism is again considered to strike the bottles collected completely by chance. On top of this, we account for the fact that bottles have finite life (measured as number of cycles made in the reuse system) due to scuffing. An inspection is made among deposited bottles that have passed the damage control, and units that bear significant signs of abrasion are rejected from further circulation. Finally, from time-to-time bottles are subject to redesign, causing the full stock of bottles and its remaining serving capacity to be discarded. Figure 1 displays a schematic outline of the reuse system.

A reuse system that only encompasses the random loss components, implicitly assumes that bottles have infinite lifetime, and no abrupt model replacement of the stock will occur. In this situation the number of servings S delivered from an initial stock of n_{start} bottles circulating the reuse system with return rate r_{rand} follows from the formula $S = n_{start} / (1-r_{rand})$, leaving a trip rate $m_{rand} = n_{start} / S = 1 / (1-r_{rand})$.

Calculating the analogous trip rate m' and back to market return rate $r' = 1 - 1/m'$ of a reuse system considering all the components described for our setup, however, requires a dynamic bookkeeping of the bottles produced and servings delivered. This is obtained via a flexible simulation setup that keeps track of the full reuse system life cycle of every single bottle.

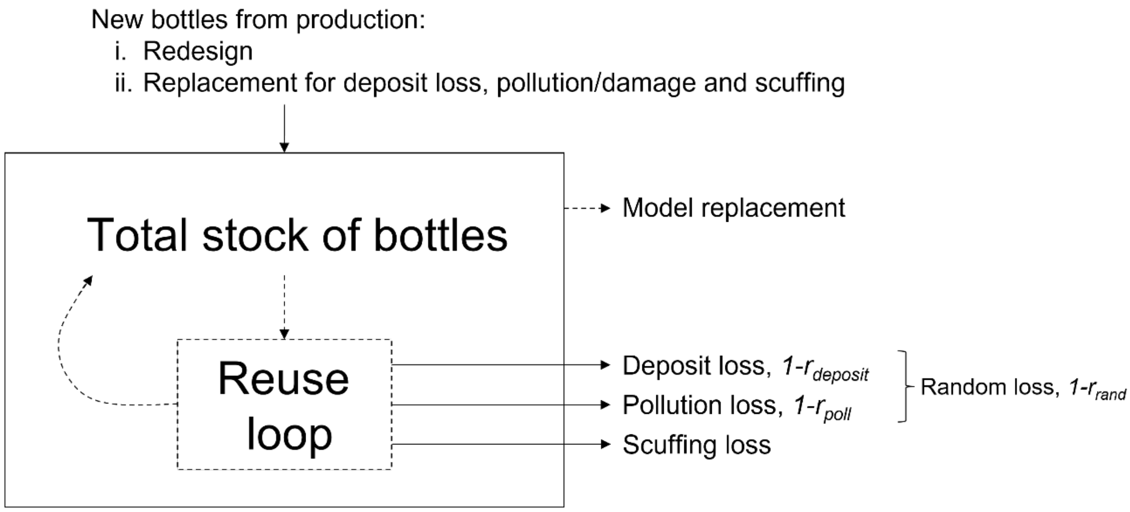

Figure 1 Schematic setup for our reuse system.

We assume that the total stock (T) of the reuse system is synthesized from four different supplies of bottles: i) those cirulating the reuse cycle (O), ii) a time-varying buffer to accommodate seasonal variation (V), iii) an extra security supply held by the beverage producers (E), and iv) a buffer to account for logistics imbalance (U). Thus, for the stock and supplies in month *i* we may write

$$T_i = O_i + V_i + E_i + U_i$$

Utilizing and interpreting information provided by Infinitum[1] on the composition of the various bottle supplies, we reformulate this expression as

$$T_i = O_i + V_i + 0.35 \cdot O_i + 0.20 \cdot O_i = 1.55 \cdot O_i + V_i$$

Furthermore, Infinitum states that the seasonal supply in July is empty, and that sales volumes in July are 60% higher than in February, ie

$$V_7 = 0$$

$$O_7 = 1.6 \cdot O_2$$

so that

$$T_7 = 1.55 \cdot O_7$$

$$T_2 = 1.55 \cdot O_2 + V_2$$

[1] Mail from Ole Faye 7 July 2023, list items 1. – 3.

And since the total stock is the same for all months ($T_2 = T_7 = T$), we deduce that

$$V_2 = 1.55 \cdot (1.6 - 1) \cdot O_2$$

so that the ratio between the number of bottles circulating the reuse cycle and the total number of bottles in stock obeys

$$c_{ratio,2} = \frac{O_2}{T_2} = \frac{1}{1.55 \cdot 1.6} = 0.40$$

$$c_{ratio,7} = \frac{O_7}{T_7} = \frac{1}{1.55} = 0.65$$

For the scuffing loss, we introduce a technical (maximal) lifetime, t_{max}, for the bottles, acknowledging that very few bottles will ever reach that age. It seems natural that the probability of being discarded due to scuffing increases with the number of trips completed in the reuse system and reaches 1 in the limit when the number of trips equals t_{max}.

Now, to identify candidates that are considered worn-out and rejected from further circulation, we implement a survival distribution approach. Let $p_i = (n_i / t_{max})^q$ be the probability of bottle *i* to be classified as worn-out due to scuffing after having completed n_i trips in the reuse system. Here, q is a shape parameter controlling the steeepness of the probablitilty distribution. As a function of n_i and for a given parameter set ($q = 10$, $t_{max} = 50$), p_i has the form shown in Figure 2.

We use the p_i values in a Bernoulli(p_i) model to decide if bottle *i* is discarded or not. This is analogous to tossing an unfair coin that has probability p_i of getting heads, meaning for instance that tossing a coin with $p_i = 0.85$ most likely will result in heads, but there is still a small probability (= 1 – 0.85 = 0.15) that the opposite side will show up. Similarly, we can decide from the p_i-s of the returned and non-polluted/non-damaged bottles in a sample which will be discarded from the stock due to scuffing.

3 Simulation study

The trip rate analysis is formulated through a simulation experiment where we consider the total stock of bottles lined up from position 1 to n_{total}. Initially, all positions are filled with brand new bottles, but over time, as bottles circulate the reuse system and produce servings, the stock will hold a mixture of ages. Using fictitious bottle positions facilitates tracking the age of every single unit.

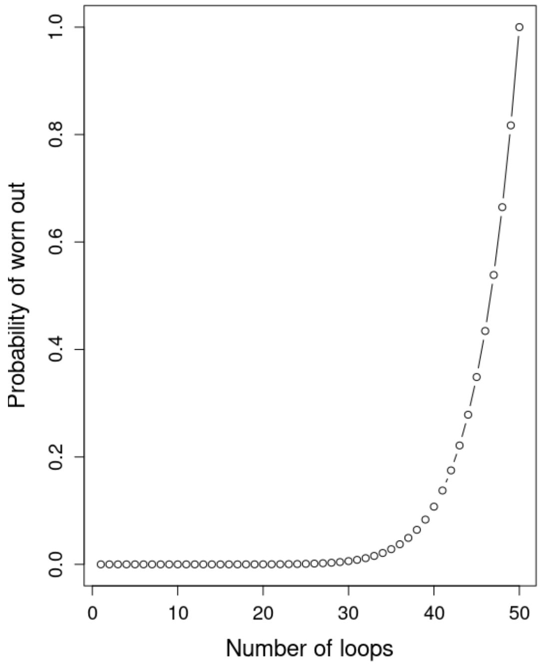

Figure 2 Probability that a bottle is discarded due to scuffing after completing a certain number of trips in the reuse system.

The experiment then starts from randomly sampling $n_{samp} \leq n_{total}$ bottles (positions) from the total stock. For this sample, delivered servings are registered along with bottle-specific cycle completions before the bottles return to the stock. Units in the sample that correspond to deposit loss, pollution/damage loss and scuffing loss are identified and rejected from further circulation. Those bottles (positions) are replaced by fresh bottles before a new sample is drawn from the total stock, cf Figure 1.

As the simulation proceeds, the age distribution of the stock will eventually stabilize. The trip rate *m'* is calculated at any time from the total number of servings delivered divided by the total number of bottles produced, automatically implementing the effect of model replacement by ignoring the remaining serving capacity of the stock at the time of a redesign.

Abandoning a closer to operational tracking approach for the bottles circulating the reuse system in favour of a stylized construction with bottle postitions has no implications for the validity and generality of the results produced by the current analysis. In particular, there is no calendar time in the simulations. Rather, we count the number of trips that bottles make in the reuse system before they are discarded. This concept connects to calendar time via the average number of cycles that is accomplished by a bottle per year, counting a cycle from the time the bottle leaves the beverage production site till it is deposited and returned through all the stages of the reuse system. The actual number varies between brands and bottle types, but estimates provided by Infinitum[2] indicate that, on average, bottles complete just above three cycles per year.

The simulation experiment defines a set of scenarios by combining certain values for parameters that describe various aspects of the reuse system and its return mechanisms. Table 1 lists those parameters along with a description of how their values have been chosen.

Table 1 Simulation parameters.

Parameter	Description
n_{total}	Total number of bottles in stock, set to 10 000 (high enough to control this part of the simulation uncertainty).
t_{design}	The number of cycles conducted in the reuse system before model replacement occurs. We consider two different situations representing brand and standard bottles, respectively. This amounts to running the simulation experiment for 6 and 18 years, corresponding to t_{design} values of 18 and 54 cycles, respectively. In a sensitivity analysis that specifically explores the effect of model replacement on the back to market return rate, we complete an additional simulation over a wide range of t_{design} values, with the remaining parameters fixed, and register corresponding r' values.
t_{max}	Technical lifetime of a bottle (in number of trips). Used as an absolute upper limit when defining the survival distribution of the bottles. Fixed at 30 trips for the current analysis.
$r_{deposit}$	Deposit return rate, taking values from two sets: {0.943, 0.891} for PET bottles and {0.921, 0.907} for glass bottles, as provided by NORSUS, based on data from Infinitum.
r_{poll}	Pollution/damage return rate, value fixed at 99% (ie 1% of the bottles are lost due to damage or pollution) inspired from an estimated value of pollution/damage loss in a former study by NR for Infinitum[3].
c_{ratio}, n_{samp}	The circulation ratio c_{ratio} is the ratio between the number of bottles sampled for circulation in each cycle of the reuse system and the total stock ($n_{samp} = c_{ratio} \cdot n_{total}$). We use this parameter as a proxy for seasonal variation, investigating values corresponding

[2] Mail from Ole Faye 8 September 2023.

[3] Rognebakke, H. and Løland, A.: Beregninger av flaskepopulasjoner, NR note SAMBA/51/10.

	to the high- and low-sales months February (c_{ratio} = 0.40) and July (c_{ratio} = 0.65), cf Problem section. For the current analysis, a mean value of c_{ratio} = 0.525 has been adopted.
$c_{wornout}$, q	The worn-out coefficient $c_{wornout}$ gives the fraction of circulated bottles that are deposited and free from pollution or damage, but still taken out from the stock due to heavy abrasion. Infinitum believes that this fraction is in the range 3-5%, but for the current simulation setup we restrict all experiments to the lower end-point value 3% only. For a specific value of t_{max}, there exists a shape parameter value q that corresponds to a chosen worn-out coefficient $c_{wornout}$. Furthermore, for each specific value of $c_{wornout}$, there exist pairs of values $\{q, t_{max}\}$ in the survival distribution that all correspond approximately to the same return rate r'. This implies that the specific value chosen for t_{max} in general is not that critical as long as the value is high enough. Preliminary investigation reveals that setting $t_{max} \geq 30$ in the current setup meets the required criteria.

In the following we summarize the various simulation experiment setups along with their results. All numbers report results from a single simulation run and thus some simulation uncertainty will be present.

For comparison of the simulated back to market trip and return rates of a reuse system with all four kinds of losses present (deposit, pollution/damage, scuffing and model replacement), the result tables also include rates corresponding to situations comprising only the random loss mechanisms (deposit and pollution/damage), or the pure deposit loss.

The survival distribution plots display the number of trips completed for bottles that are discarded from the reuse system due to scuffing, ie the age of the bottles at the time when scuffing ends their lives.

We split the simulation study into two main experiments involving PET and glass bottles, respectively. For each bottle material we consider two different bottle sizes with their individual deposit return rate $r_{deposit}$. Additionally, for the 1.5 litres PET bottles we elaborate on the main results by giving an example of the individual survival distributions for all sources of loss. Also, we include a sensitivity analysis that illustrates how the back to market return rate depends on the time to model replacement for a wide range of t_{design} values.

For reference, the most central parameters and quantities discussed in the text but not mentioned in Table 1 are summarised in Table 5 at the end of the document.

Main experiment 1: PET bottles

Parameter set:

- $r_{deposit} \in \{0.943, 0.891\}$
- $t_{design} \in \{18, 54\}$
- $c_{wornout} = 3\%$

Results are given in Table 2, and corresponding survival distributions for the loss due to scuffing are displayed in Figure 3.

Table 2 Setup and results from simulation experiment 1.

Type	t_{design}	q	$c_{wornout}$	m'	r'	m_{rand}	r_{rand}	$m_{deposit}$	$r_{deposit}$
PET 1.5	18	2.8	2.6%	5.5	81.9%	15.1	93.4%	17.5	94.3%
	54	3.5	3.2%	8.2	87.7%	15.1	93.4%	17.5	94.3%
PET 0.5	18	2.2	3.2%	4.2	76.2%	8.5	88.2%	9.2	89.1%
	54	2.6	3.0%	5.8	82.6%	8.5	88.2%	9.2	89.1%

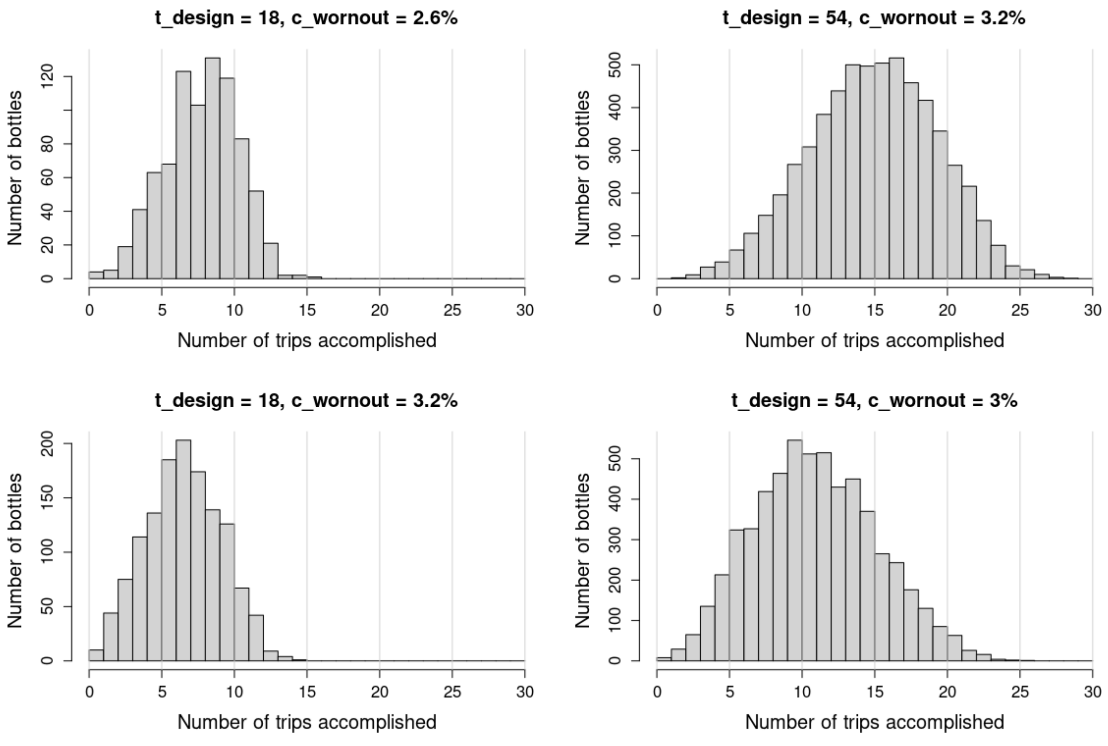

Figure 3 Survival distributions for the loss due to scuffing for simulation experiment 1. Upper panels represent 1.5 l PET bottles, whereas results for 0.5 l PET bottles are shown in the lower panels. All simulations correspond to an approximate worn-out ratio of 3%.

Main experiment 2: Glass bottles

Parameter set:

- $r_{deposit} \in \{0.921, 0.907\}$
- $t_{design} \in \{18, 54\}$
- $c_{wornout} = 3\%$

Results are given in Table 3, and corresponding survival distributions for the loss due to scuffing are displayed in Figure 4.

Table 3 Setup and results from simulation experiment 2.

Type	t_{design}	q	$c_{wornout}$	m'	r'	m_{rand}	r_{rand}	$m_{deposit}$	$r_{deposit}$
Glass 0.5	18	2.5	3.1%	4.9	79.5%	11.3	91.2%	12.7	92.1%
	54	3.3	3.1%	7.1	85.9%	11.3	91.2%	12.7	92.1%
Glass 0.33	18	2.3	2.8%	4.5	77.9%	9.8	89.8%	10.8	90.7%
	54	2.8	2.9%	6.3	84.2%	9.8	89.8%	10.8	90.7%

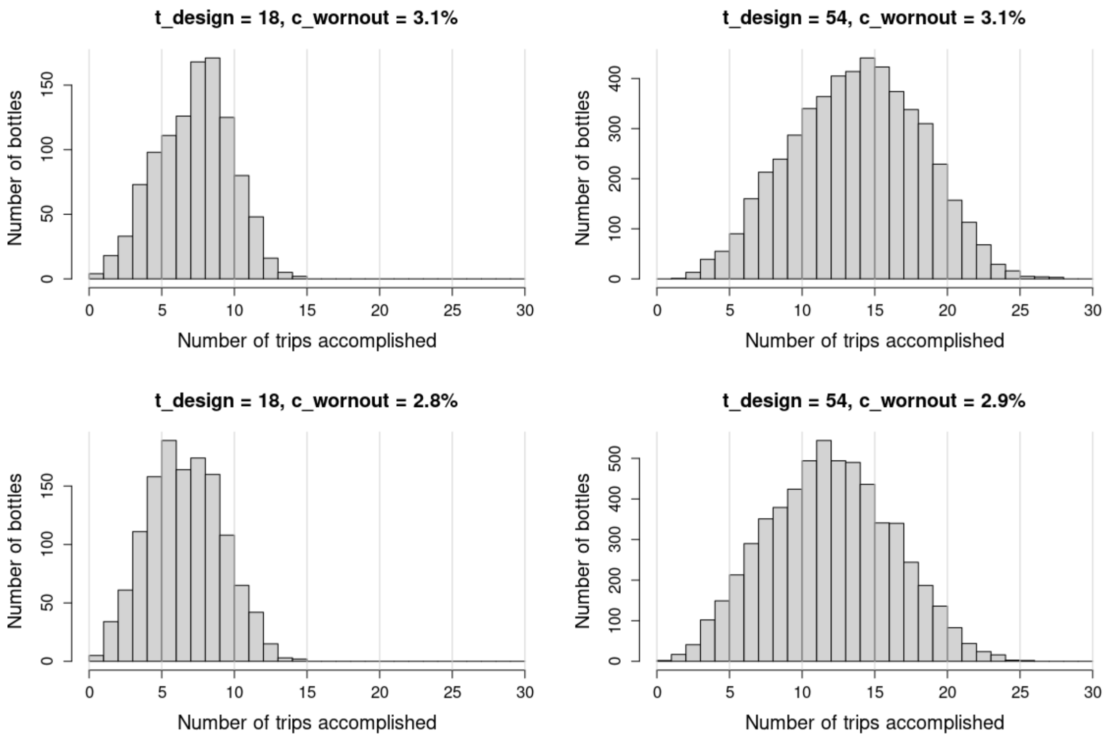

Figure 4 Survival distributions for the loss due to scuffing for simulation experiment 2. Upper panels represent 0.5 l glass bottles, whereas results for 0.33 l glass bottles are shown in the lower panels. All simulations correspond to an approximate worn-out ratio of 3%.

Additional experiments

Similar to the scuffing survival distribution plots displayed in Figures 3 – 4, we can generate plots for the other sources of loss as well: deposit, pollution/damage and the collective loss also including scuffing. This is illustrated in Figures 5 – 6 for main experiment 1 under the 1.5 l PET bottle setup using t_{design} = 18 and t_{design} = 54, respectively. All distributions show cumulative bottle lifetimes over the full simulation period up to t_{design}.

The immaturity of the simulation with the shortest time to model replacement is evident from the fact that no bottles can complete more than t_{design} = 18 trips which is well below their technical lifetime of t_{max} = 30 cycles. We also recognize the striking difference between the shape of the scuffing loss distribution and the distributions for the random loss components (deposit and pollution/damage loss). The total loss distribution inherits properties from all three sources but is heavily influenced from the weighting implied by the number of bottles attributed to each category.

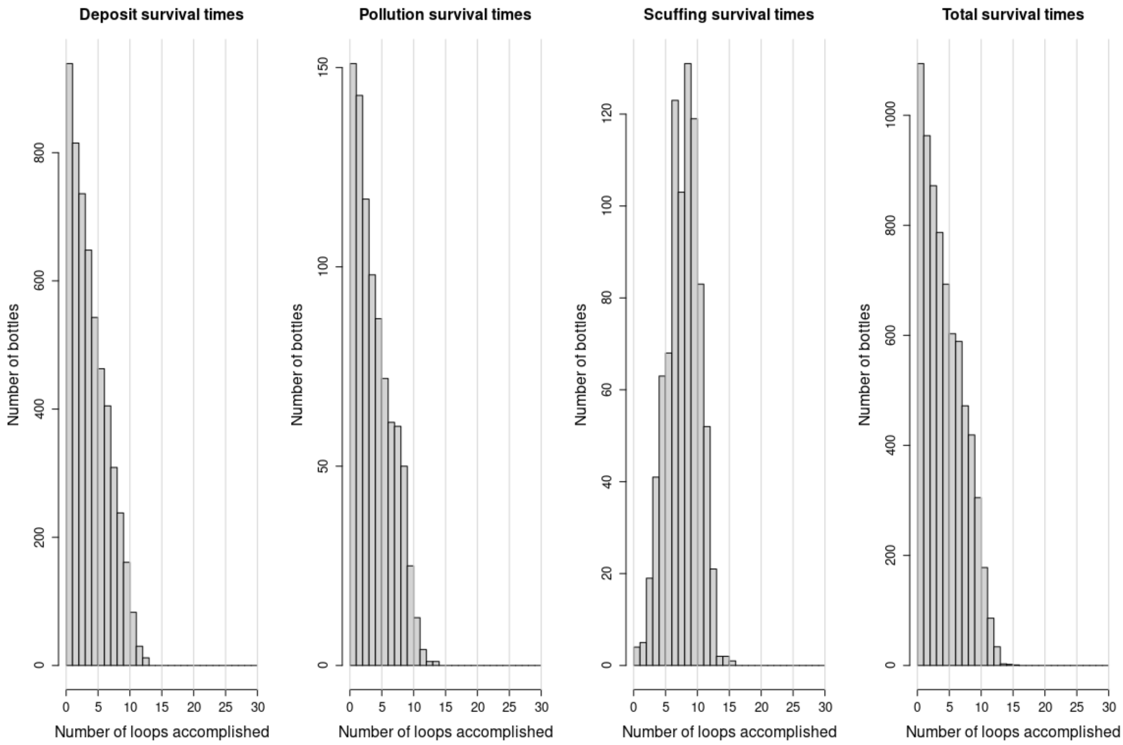

Figure 5 Survival distributions for the individual loss mechanisms involved in the reuse system for main experiment 1 under the PET 1.5 l PET bottle setup with t_{design} = 18.

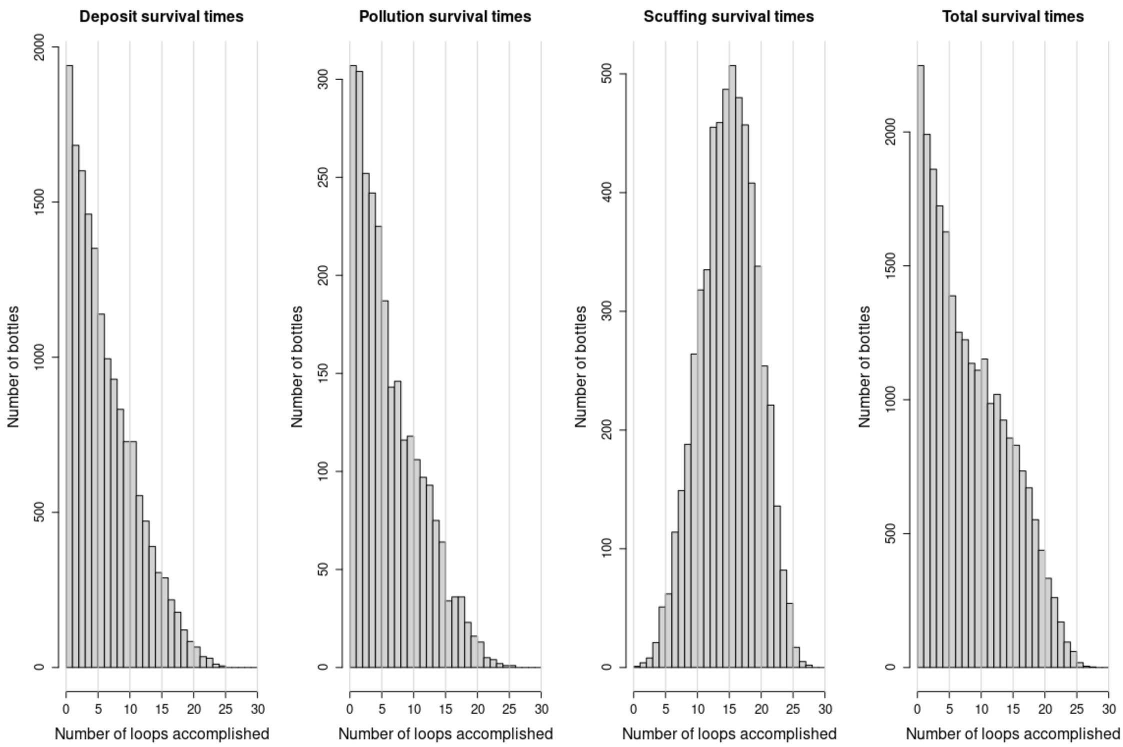

Figure 6 Survival distributions for the individual loss mechanisms involved in the reuse system for main experiment 1 under the PET 1.5 l PET bottle setup with t_{design} = 54.

Finally, it is also informative to verify how the back to market return rate *r'* varies with time to model replacement, t_{design}. Intuitively, the impact of discarding the serving potential of the remaining stock at the time of a redesign will diminish as t_{design} grows large enough.

We investigate the connection between *r'* and t_{design} under the 1.5 l PET bottle parameter setup of main experiment 1. The results are listed in Table 4 and further visualized in Figure 7.

Table 4 Trip and return rates as a function of time to model replacement, keeping the worn-out fraction at approximately 3%.

t_{design}	$c_{wornout}$	m'	r'	m_{rand}	r_{rand}	$m_{deposit}$	$r_{deposit}$
2	3.0%	1.0	-4.0%	15.1	93.4%	17.5	94.3%
3	3.0%	1.4	27.9%	15.1	93.4%	17.5	94.3%
5	3.1%	2.2	53.6%	15.1	93.4%	17.5	94.3%
7	2.8%	2.8	64.7%	15.1	93.4%	17.5	94.3%
10	3.1%	3.7	73.1%	15.1	93.4%	17.5	94.3%
15	3.3%	4.8	79.4%	15.1	93.4%	17.5	94.3%
20	3.5%	5.8	82.7%	15.1	93.4%	17.5	94.3%
30	3.3%	7.1	86.0%	15.1	93.4%	17.5	94.3%
50	3.2%	8.0	87.5%	15.1	93.4%	17.5	94.3%

70	3.2%	8.7	88.5%	15.1	93.4%	17.5	94.3%
100	2.8%	9.2	89.2%	15.1	93.4%	17.5	94.3%
200	3.0%	9.8	89.8%	15.1	93.4%	17.5	94.3%

Bearing in mind that, on average, bottles complete slightly more than three cycles in the reuse system per year, we see that it takes roughly five years (15 cycles) for the back to market return rate to reach 80%, and another 28(!) years (100 cycles in total) to more or less stabilize at its final value close to 90%. From this we draw the conclusion that a model replacement in early years after the launch of a new design significantly reduces the overall return rate of the system (at least for the current parameter setup).

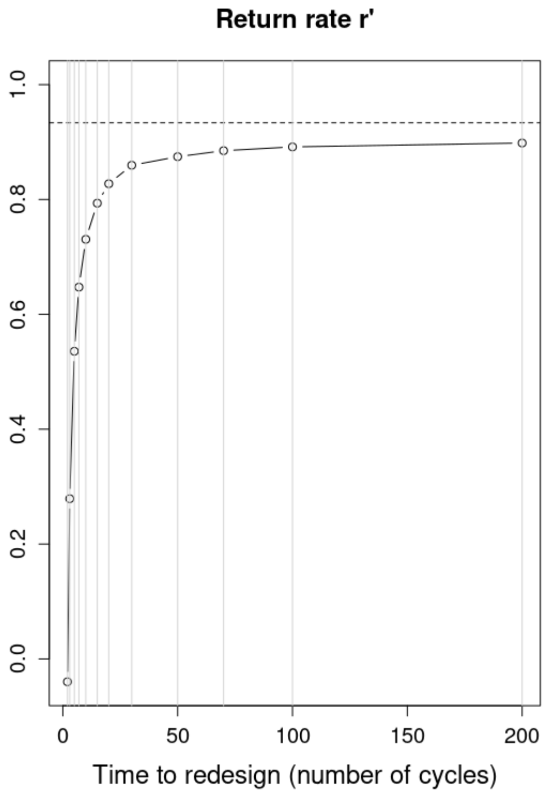

Figure 7 Return rate r' as a function of time to model replacement.

Table 5 List of survival distribution parameters and output quantities.

Parameter	Description
n_i	Survival distribution variable representing the number of trips that bottle *i* has completed in the reuse system at a certain point.
q	Survival distribution shape parameter controlling the steepness of the probability distribution.
p_i	Probability of bottle *i* to be classified as worn-out due to scuffing after having completed n_i trips in the reuse system.
$r_{deposit}$	Deposit return rate (where deposit is assumed to act randomly among all bottles circulating the reuse system regardless of age and condition).
$m_{deposit}$	Deposit trip rate, $m_{deposit} = 1 / (1 - r_{deposit})$.
r_{rand}	Compound return rate from the random loss mechanisms (deposit and damage/pollution), $r_{rand} = r_{deposit} \cdot r_{poll}$.
m_{rand}	Compund random loss trip rate, $m_{rand} = 1 / (1 - r_{rand})$.
m'	Back to market trip rate. m' is calculated from the total number of servings delivered divided by the total number of bottles produced at a certain point.
r'	Back to market return rate, $r' = 1 - 1/m'$.